Wire span simulator

Ben Stabley - CIS122 - Lab 9 - Final Submission of ongoing lab project

Background

When a flexible chain, rope, or wire with uniform weight over its length is suspended from 2 points, the shape it produces under the effect of gravity is called a "catenary".

For overhead electrical wires, the catenary shape can be used to calculate important safety clearances. Especially in high voltage transmission (110kV and above) when electricity can more easily jump the gap to vegetation, towers, or other wires, wires must maintain clearance under a number of conditions. Inadequate clearance is a hazard for power loss and possibly fires.

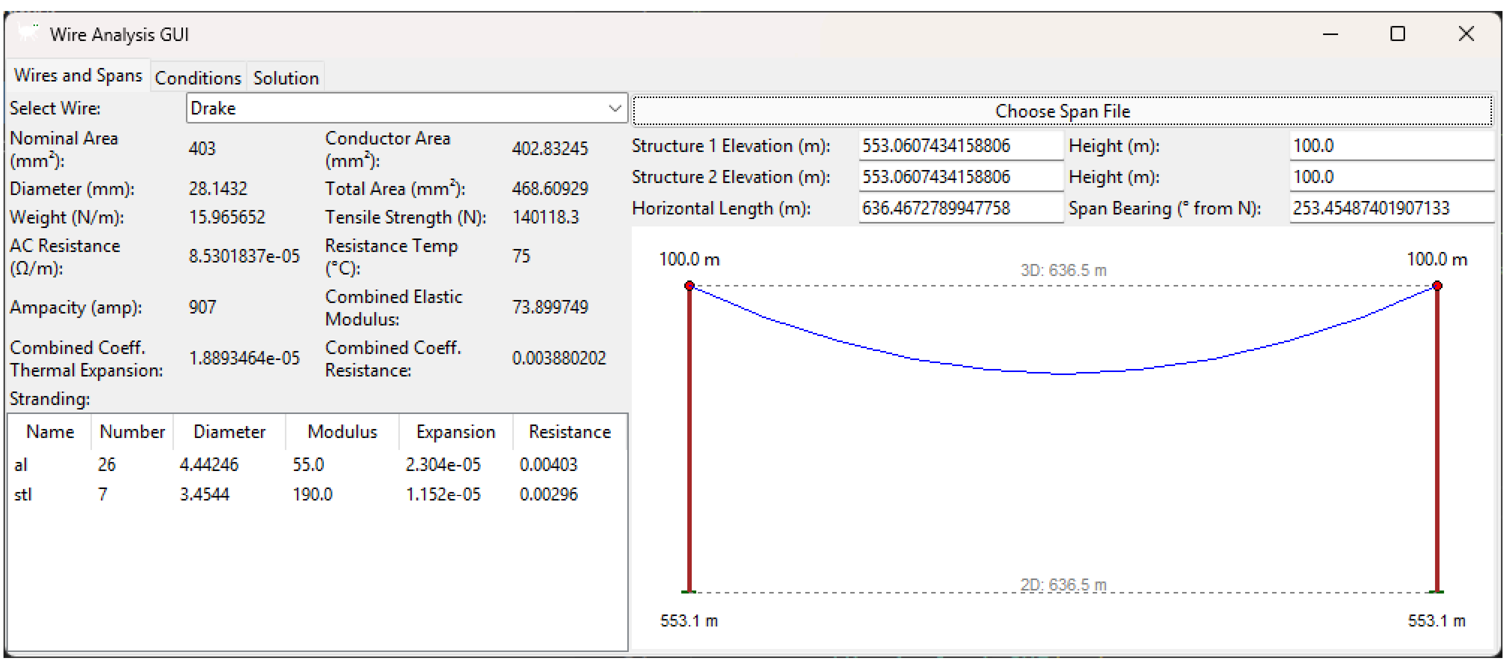
Though made of thick aluminum and steel, wires will naturally elongate from their original manufactured length due to initial settlement and age-related deformation. Additionally, wires elongate as a function of temperature--both environmental temperature and heat generated during operation due to electrical resistance. Other significant environmental factors include wind, water, ice, and varnish build-up which, when present, add to the overall weight per unit-length of the wire.

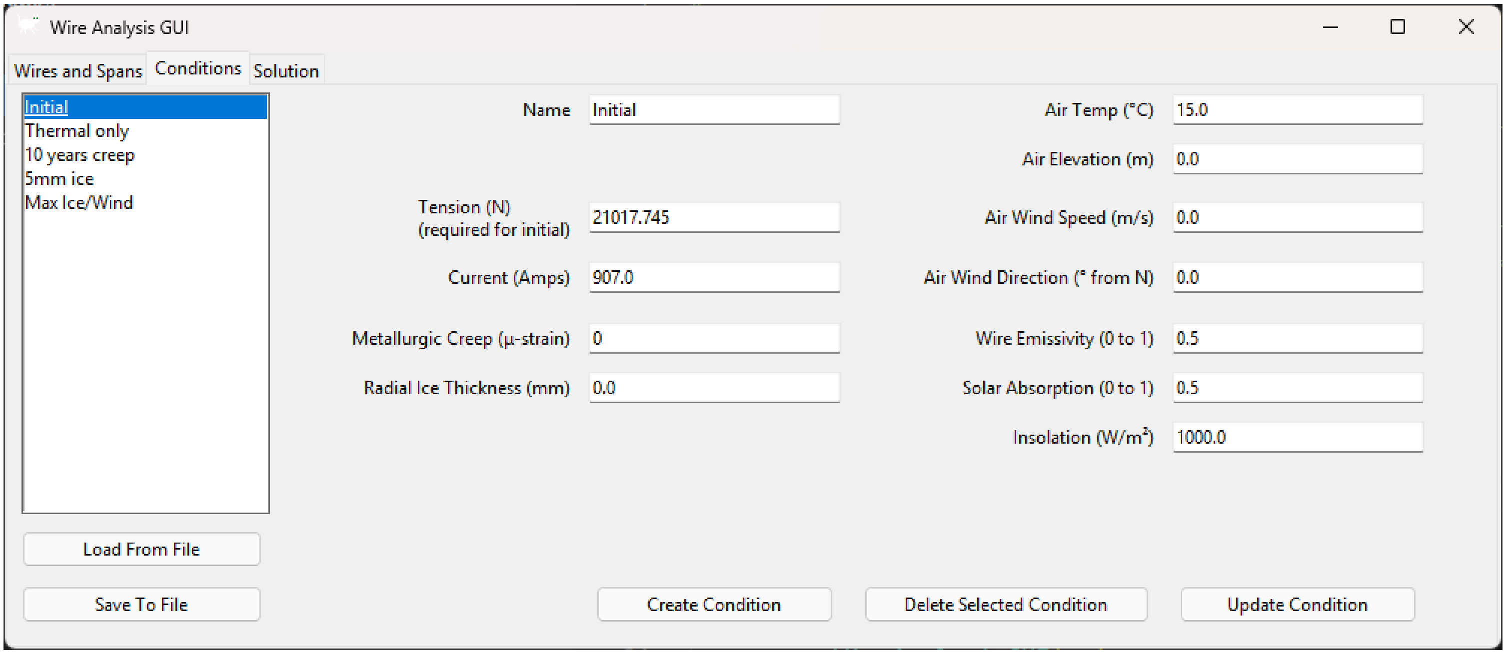
Wire elongation changes the catenary shape, causing the lowest point of the wire to drop relative to the straight-line between wire attachment points. The line between attachments is known as the "span" and the drop is known as "sag".

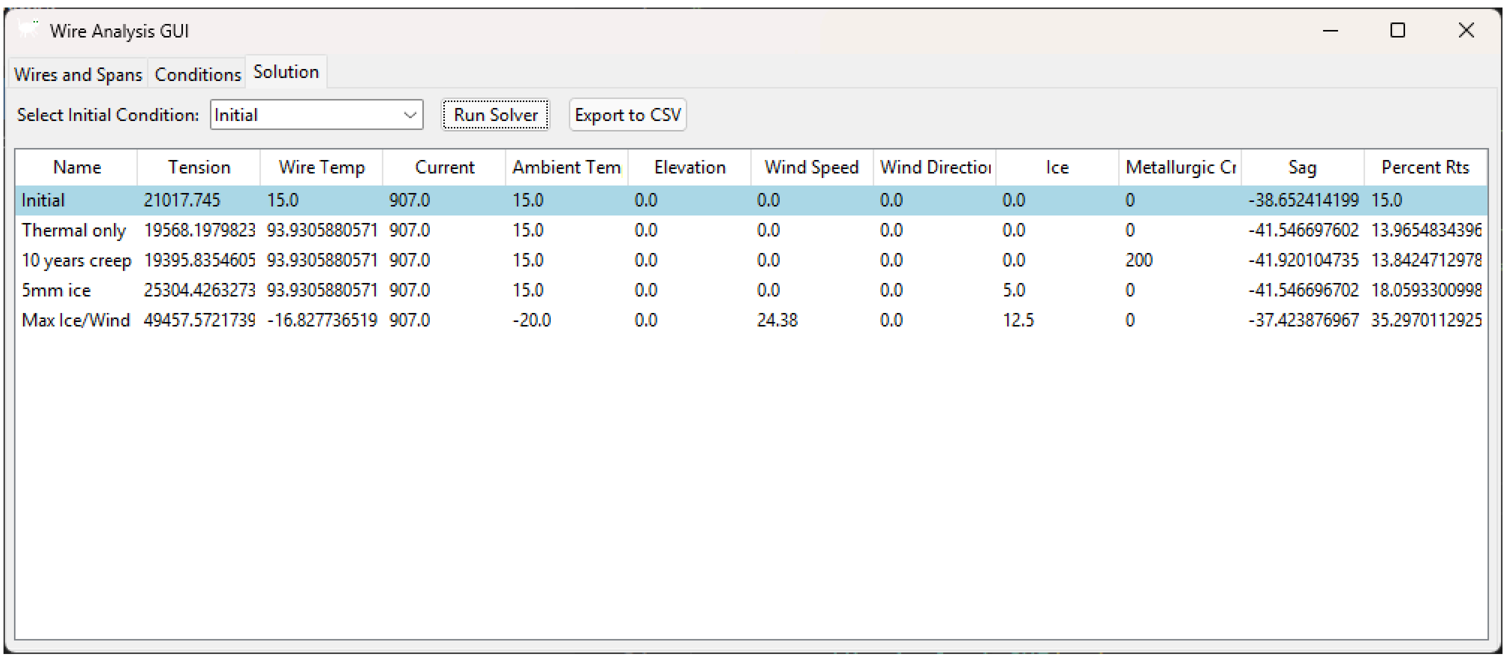
Goals

1. Use the catenary equation to calculate wire sag and clearance for level and inclined spans.
2. Use real wire specifications for calculations.
3. Account for temperature elongation and icing. Possibly wind.
4. Expand simulation to deal with a series of spans such as might be found in an electrical network.
5. Produce visualizations, possibly interactive 3D.

Trial Run







Wire Analysis GUI Program

Launch program

Change to the root code folder and run `python main.py`.

Usage

There are 3 tabs in the "Wire Analysis GUI": Wires and Spans, Conditions, and Solution. Each one configures a different part of the overall system.

All units are SI units (metric).

Wires and spans tab

Wires and Spans lets you configure the equipment component by choosing a wire type from those provided (all are standard ACSR wires) and configuring what is known as the "ruling span". A ruling span is a length-weighted average span that represents a long multi-span run of an electrical circuit. You may enter the structure and span parameters directly or load a circuit from a CSV file.

I have provided 2 example circuits. simple_span.csv is just 1 span with the very minimal file requirements: an X column with 2 structures in some abstract coordinate system. BPA_TROUTDALE-DALLES_500KV.csv is an actual transmission circuit that runs from The Dalles, through the mountains north of Mt Hood, near Sandy, and ultimately ends up near Barton. It contains structure positions in the "Web Mercator" coordinate reference system, which measures position as meters from the Prime Meridian (X) and from the Equator (Y). Elevation values for the structure base (Z) are specified as meters above sea-level.

Conditions tab

Conditions allows you to configure the operating and environmental conditions of the wire. These could be field-measured values or theoretical maximums specified by regulator bodies. Operating conditions are things like the current flow in the wire or the plastic elongation (creep) experienced by the wire due to age. Environmental conditions are air and weather, solar, and wire surface conditions (eg dirtiness).

You may create new conditions by completing the form and pressing "Create Condition". You can modify a condition by selecting it, editing the form values, and pressing "Update Condition". Finally, you can delete a condition by selecting it and pressing "Delete Selected Condition". Each condition must have a unique name. "Tension" is really only required for the condition you decide to use as the "initial condition" (see Solution tab). If you select a wire type other than "Drake", you may want to modify the initial condition's tension to be about 10-20% of that wire's tensile strength.

You may also save your configured conditions to disk in a JSON format and load them later. Of course, you may also edit or add conditions via editing the JSON file though this is beyond the scope of this document. I have provided a set of 4 conditions in conditions.json for you to try. The condition named "Initial" is intended to be used as the initial condition for the solution tab.

Solution tab

The solution tab allows you to choose an initial condition, "solve" the remaining conditions, view the results, and optionally export them to CSV.

An "initial condition" is essentially a starting point for the circuit. All other conditions represent events that happen to the ruling span after the initial condition. The initial condition could be when the wire is first installed, before power is sent through, or some other "everyday" operating condition. In the sample conditions I have provided, the initial condition is intended as the former (first installed).

The program will solve each non-initial condition first for the wire temp based on thermal variables (current flow, weather, etc). Then it will solve for the horizontal tension based on thermal, creep, and weight-induced elongation of the wire. Select properties of the solved condition along with the sag and the final tension's percentage of the wires max rated tensile strength (RTS or breaking strength) are shown in the table.

Sag and %RTS are critical for 2 reasons:

1. Excessive sag can cause the wire to come close enough to vegetation or buildings that an arc forms, causing power outage and potentially fire.
2. If the wire experiences tension near its %RTS, the wire could physically break, also leading to power outage and fire. This is especially important as the wire ages.

Author comments about code

Two main points:

First, not all the code has full docstrings. This is especially true of the gui code, which was largely generated by ChatGPT to create a skeleton. I then modified it to work with the view-model paradigm as well as tweak the labels, widget positions, etc. Most functions and types have docstrings, but they are not necessarily in Google's documentation format.

Second, although the program produces calculations, I don't believe the solved tension (and therefore sag and %RTS) are actually correct. I had good success with the catenary shape equations that you saw in earlier labs, and also had success with the thermal solving portion of the solver. However, I struggled with the tension solver as my (many) sources didn't really specify a clear algorithm.

Overall, I actually spent a *lot* of time in the middle of the quarter trying to understand the tension-solving methods and got behind on the other aspects of the project, such as code documentation and GUI/UI development. Despite this, I think I did achieve most of the goals I set for this program at the beginning of the quarter.

Sources

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13. A New Computer Model of ACSR Conductors. https://ieeexplore.ieee.org/document/4111976
14. BPA transmission line GIS data from: https://atlas.eia.gov/

Appendix: Equations

Mostly for Ben's reference.

From Douglass 2016 [1]

symbol	meaning
S	span length
L	catenary conductor length
D	catenary low point sag distance (vertical)
$\sinh^{-1}$	arcsinh, etc. *not* $\frac{1}{\sinh}$
H	tension, N
w	conductor unit weight, kg/m
E	modulus of elasticity
A	conductor cross-sectional area, m²
α	coefficient of thermal expansion
σ	stress = $\frac{H}{A}$, MPa
ϵ	strain, parts-per-million ($\frac{1}{1e6}$) or "microstrains"

- eq 3. Total conductor length (left and right) relative to catenary low point.

$$L(x) = \frac{H}{w} \cdot \sinh\left(\frac{w \cdot x}{H}\right)$$

- eq 7. "Slack" = conductor length - span length:

$$slack = L - S$$

- eq 9. Conductor position (left and right) relative to catenary low point. $y > 0$

$$y(x) = \frac{H}{w}\left(\cosh\left(\frac{w \cdot x}{H}\right) - 1\right)$$

- eq 10. Catenary low point relative to support. h is relative structure heigh difference. Note h is relative to support in question. h<0 indicates the conductor is declined from support; h>0 indicates conductor is inclined from support.

$$\begin{aligned} C &= \frac{H}{w} = \text{catenary constant} \\ S_h &= \frac{S}{2} = \text{half span length} \\ X_D &= S_h - C\sinh^{-1}\left(\frac{h}{2C\sinh\left(\frac{S_h}{C}\right)}\right) \end{aligned}$$

- eq 13. Linear elastic strain

$$\begin{aligned} \Delta L &= L - L_{ref} \\ \Delta H &= H - H_{ref} \\ \Delta L &= L_{ref}\left(\frac{\Delta H}{E \cdot A}\right) \\ L &= L_{ref}\left(1 + \frac{H - H_{ref}}{E \cdot A}\right) \\ L &= L_{ref}\left(1 + \epsilon_\sigma\right) \end{aligned}$$

- eq 14. Linear thermal strain

$$\begin{aligned} \Delta L &= L - L_{ref} \\ \Delta T &= T - T_{ref} \\ \Delta L &= L_{ref} \cdot \alpha \cdot \Delta T \\ L &= L_{ref}\left(1 + \alpha\left(T - T_{ref}\right)\right) \end{aligned}$$

- eq 19. Combined elastic modulus for non-homogeneous conductors (ie weighted average of component E, weighted by proportion of cross-sectional area)

$$E_{AS} = E_A \cdot \frac{A_A}{A_{AS}} + E_S \cdot \frac{A_S}{A_{AS}}$$

- eq 20. Combined coefficient of thermal expansion (ie weighted average of component α, weighted by proportional $E \cdot A$

$$\alpha_{AS} = \alpha_A\left(\frac{E_A}{E_{AS}} \cdot \frac{A_A}{A_{AS}}\right) + \alpha_S\left(\frac{E_S}{E_{AS}} \cdot \frac{A_S}{A_{AS}}\right) = \frac{(\alpha_A E_A A_A + \alpha_S E_S A_S)}{E_{AS} A_{AS}}$$

- Pg 51. Creep elongation with time. σ expressed in MPa and t in hours. Note that K here is a constant from IEEE for different conductors based on casting method and strand numbers. I don't know why Douglass chose 1.36 (hot-rolled 7 strands of AL), but maybe because it's the most conservative (of the small table listed by PLS-CADD)?

$$\begin{aligned} \epsilon_C &= K \cdot \sigma^{1.3} \cdot t^{0.16} \\ K &= 1.36 \end{aligned}$$

From Slegers 2011 [2]

symbol	meaning
l	span length
L	catenary conductor length
S	catenary low point sag distance (vertical)
D	conductor diameter
$\sinh^{-1}$	arcsinh, etc. *not* $\frac{1}{\sinh}$
H	tension
w	conductor unit weight
E	modulus of elasticity
A	conductor cross-sectional area
α	coefficient of thermal expansion
σ	stress = $\frac{H}{A}$
ϵ	strain, unitless, parts-per-million ($\frac{1}{1e6}$) or "microstrains"

- eq 1.6. Conductor length under stress. Similar to eq 13 in Douglass, but with creep.

$$\begin{aligned} L_\sigma &= L \cdot (1 + \epsilon_\sigma + \epsilon_C) \\ L &= \text{length with } \textbf{no} \text{ stress} \\ \epsilon_\sigma &= \frac{H}{E \cdot A} \\ \epsilon_C &= \text{creep plastic deformation} \end{aligned}$$

- eq 1.7. Conductor length under stress and temperature. Again, similar to eq 14 in Douglass, but combines stress and temperature elongation. Slegers shows using w_0 and $w_{condition}$ to make a (numerically) solvable equation for H with $L_{condition}(H) = L_0(H)$.

$$\begin{aligned} L &= L_0 \cdot (1 + \alpha \Delta T) \cdot (1 + \epsilon_\sigma + \epsilon_C) \\ L &= L_0 \cdot (1 + \alpha(T - T_0)) \cdot \left(1 + \frac{H - H_0}{E \cdot A} + \epsilon_C\right) \end{aligned}$$

- Ice volume. Below is my simplification using diameter D instead of radius R. Slegers adds stupid unit conversions to the equation. Total ice weight is then $w = v \cdot \rho$ where ρ is density.

$$\begin{aligned} v &= \pi \left((R_w + R_i)^2 - R_w^2 \right) \\ &= R_i \pi \left(R_i + 2R_w \right) \end{aligned}$$

- Heat Balance: $Q_C + Q_R = Q_S + Q_{EM}$
  Slegers uses this to solve for ampacity. But I'm numerically solving with newton's method to find the wire temp at a particular current.
    - Convective heat loss. Does *not* specify what constitutes "low" and "high" speed wind. Provides table for properties of air at various temps, but does not specify pressure and moisture of air.

$$Q_C = \pi \lambda Nu (T - T_a)$$

$$\begin{aligned} Re &= \frac{vD\gamma}{\eta}, \text{reynolds number} \\ Nu &= 0.32 + 0.43 Re^{0.52}, \text{for low speed winds} \\ Nu &= 0.24 Re^{0.6}, \text{for high speed winds} \\ \lambda &= \text{thermal conductivity of air} \end{aligned}$$

- Radiant heat loss. T in °K. k_s constant

$$Q_R = k_s k_e D\pi \left(T^4 - T_a^4\right)$$

$$\begin{aligned} k_s &= \text{Stephan-Boltzmann constant for black body radiation} \\ k_e &= \text{emission coefficient.} \approx 0 \text{ for new, 0.5 - 1 for dirty/oxidized, usually} \approx 0.6 \\ T_a &= \text{ambient temperature} \end{aligned}$$

- Solar heat gain (insolation).

$$Q_S = D k_a Q_{SH}$$

$$\begin{aligned} k_a &= \text{absorption coefficient, unitless. usually around 0.5} \\ Q_{SH} &= \text{standard solar radiation. varies geographically 850 - 1350} \end{aligned}$$

- Resistive heat gain (AC losses). Resistance for AC.

$$Q_{EM} = I^2 R_T$$

$$\begin{aligned} R_T &= R_{ref}\left(1 + \alpha_R \left(T - T_{ref}\right)\right) \\ \alpha_R &= \text{temperature coefficient of resistance} \end{aligned}$$

From Alexiou [3]

- Vertical conductor sag position y as a function of distance from structure. (x_c, y_c) = catenary sag low point, S = span length, a = catenary constant, h = relative structure height difference. For some reason Alexiou substitutes an exponential expression for $\sinh$ but they appear to be equivalent.

$$\begin{aligned} y(x) &= y_c + a\left(\cosh\left(\frac{x - x_c}{a}\right) - 1\right) \\ x_c &= \frac{S}{2} + a \sinh^{-1}\left(\frac{h\, \mathrm{e}^{\frac{S}{2a}}}{a\left(1 - \mathrm{e}^{\frac{S}{a}}\right)}\right) \\ y_c &= -a\left(\cosh\left(\frac{x_c}{a}\right) - 1\right) \end{aligned}$$